J. Fluid Mech. (1981), *vol.* 108, *pp.* 147–157
Printed in Great Britain

Newtonian flow theory for slender bodies in a dusty gas

By R. M. BARRON AND J. T. WILEY
Department of Mathematics, University of Windsor, Ontario, Canada N9B 3P4

(Received 9 January 1980 and in revised form 15 September 1980)

Hypersonic small-disturbance theory is extended to consider the problem of dusty-gas flow past thin two-dimensional bodies. The mass fraction of suspended particles is assumed to be sufficiently large that the two-way interaction between particle phase and gas phase must be considered. The system of eight governing equations is further reduced by considering the Newtonian approximation $\gamma \rightarrow 1$ and $M_\infty \rightarrow \infty$. The Newtonian theory up to second order is studied and the equations are solved for the case of a thin wedge at zero angle of attack. Expressions for the streamlines, dust-particle paths, shock-wave location and all flow variables are obtained. It is seen that the presence of the dust increases the pressure along the wedge surface and tends to bend the shock wave towards the body surface. Other effects of the interaction of the two phases are also discussed.

1. Introduction

The problem of high-speed flow of a gas containing small particles past a solid body has received considerable attention since the work of Probstein & Fassio (1970). This work, which considered flow past thin wedges and cones and also spherical and cylindrical nose geometries, was followed by the investigations of Waldman & Reinecke (1971) for conical and spherical shock layers, Spurk & Gerber (1972) who studied flow past thin power-law bodies, and others. Much of this work has been extended by Peddieson & Lyu (1973, 1974) and Peddieson (1975). Some corrections to previous results have been provided by Barron & Wiley (1979). In all of these works it is assumed that a volume element of the suspended phase, whose dimensions are small compared to the characteristic dimensions of the shock layer, contains a large enough number of particles to allow the formation of meaningful averages of the particle properties within the volume element. The volume element is then treated mathematically as a differential element and the averages are treated as continuous variables, i.e. the particle cloud is treated as a continuum. This approach is retained in the present investigation. All the above works also assume that the mass fraction of suspended particles is so small that the presence of the particles does not significantly affect the motion of the gas. This reduces the full problem to that of obtaining the motion of the particles as they move through a known gas field. However, Peddieson (1975) has shown that the decrease in particle-phase normal velocity is accompanied by a compression of the particle phase, and this compression increases in severity as the interphase momentum transfer coefficient increases. This is important in that it shows that the particle-phase mass fraction may be negligible in the free stream but non-negligible near the body surface. When this situation arises, the results of the above works become unreliable. The present paper does not require this assumption, and hence the effects of the two-way interaction between the particle phase and gas phase are considered.

In this paper the well-known hypersonic small-disturbance theory of Van Dyke (1954) is extended to the dusty-gas flow problem. The system of equations is further reduced by considering the Newtonian approximation $\gamma \to 1$ and $M_\infty \to \infty$. The Newtonian theory up to second order is studied and the equations are solved analytically for the case of a thin wedge at zero angle of attack. The results are illustrated graphically and compared with previous results.

2. Governing equations

Consider two-dimensional steady two-phase flow past a symmetric body at zero angle of attack. The basic formulation is that reviewed by Marble (1970). Treating the fluid phase and the dust particle phase as two interacting continua, the steady dusty-gas flow is governed by

$$\nabla.(\rho'\mathbf{u}') = 0, \quad \rho(\mathbf{u}'.\nabla)\mathbf{u}' = -\nabla p' + \mathbf{F}', \tag{1),(2}$$

$$c_v R^{-1}\rho'\mathbf{u}'.\nabla(p'/\rho') + p'\nabla.\mathbf{u}' = Q' + \mathbf{F}'.(\mathbf{v}'-\mathbf{u}'), \tag{3}$$

$$\nabla.(N'v') = 0, \quad mN(\mathbf{v}'.\nabla)\mathbf{v}' = -\mathbf{F}', \tag{4), (5}$$

$$mcN'\mathbf{v}'.\nabla T'_p = -Q'. \tag{6}$$

In these equations, ρ', $\mathbf{u}'$ and p' are the gas mass density, velocity field and pressure respectively, N', $\mathbf{v}'$ and T'_p are the dust-particle number density, velocity field, and temperature respectively. Also m is the mass of each individual dust particle (assumed to be the same for all dust particles), c is the specific heat of the dust, c_v is the specific heat of the gas at constant volume, and R is the gas constant given by Charles' law for the perfect gas under consideration, i.e.

$$R = p'/(\rho' T'), \tag{7}$$

T' being the gas temperature. Finally, the quantities $\mathbf{F}'$ and Q' represent the force exerted upon a unit volume of gas by the particles and the heat transferred from particle phase to gas phase. Following Marble (1970), we assume the appropriate forms for $\mathbf{F}'$ and Q' are

$$\mathbf{F}' = kN'(\mathbf{v}'-\mathbf{u}'), \quad Q' = DN'(T'_p - T') = DN'\{T'_p - (p'/R\rho')\}. \tag{8}$$

The boundary conditions associated with system of equations (1)–(6) are the jump conditions at the shock wave (assumed to be attached), the conditions far upstream, and the requirement that the relative normal gas velocity component at the body surface vanishes. In addition, it is assumed that the variables describing the dust-particle flow are continuous across the shock (cf. Carrier 1958).

Shock conditions: At the shock, defined by $S'(x', y', z') = 0$, we have

$$\left.\begin{aligned} &[\mathbf{u}']\alpha\nabla S', \\ &[\rho' u'_n] = [\rho' u'^2_n + p'] = [\tfrac{1}{2}u'^2_n + \gamma p'/(\gamma-1)\rho'] = 0, \\ &\mathbf{v}' = \mathbf{U}_\infty \mathbf{i}, \quad N' = N_\infty, \quad T'_p = T_\infty. \end{aligned}\right\} \tag{9}$$

Upstream conditions:

$$\mathbf{u}' \to \mathbf{U}_\infty \mathbf{i}, \quad p' \to p_\infty, \quad \rho' \to \rho_\infty \quad \text{as} \quad x \to -\infty. \tag{10}$$

Surface condition: Let the surface be defined by $B'(x', y', z') = 0$. Then, the condition at the body surface is

$$\mathbf{u}' . \nabla B' = 0. \tag{11}$$

3. Thin bodies and the Newtonian approximation

For two-dimensional flows the hypersonic small-disturbance theory of Van Dyke (1954) for clean gases can be extended to dusty gas flows as follows. Let τ be a measure of the thinness of the body surface and define undashed variables which are $O(1)$ as $\tau \to 0$, $M_\infty \to \infty$ such that $M_\infty \tau = O(1)$ as:

$$\left.\begin{aligned} &x = x', \quad y = y'/\tau, \quad u_1' = U_\infty\{1 + \tau^2 u_1 + \ldots\}, \quad u_2' = U_\infty \tau u_2 + \ldots, \\ &\rho' = \rho_\infty \rho + \ldots, \quad p' = \gamma p_\infty M_\infty^2 \tau^2 p + \ldots, \quad N' = N_\infty N + \ldots, \\ &v_1' = U_\infty\{1 + \tau^2 v_1 + \ldots\}, \quad v_2' = U_\infty \tau v_2 + \ldots, \quad T_p' = T_\infty T_p + \ldots, \end{aligned}\right\} \tag{12}$$

where $\mathbf{u}' = (u_1', u_2')$, $\mathbf{v}' = (v_1', v_2')$ and M_∞ is the free-stream Mach number. Furthermore, assuming the body surface to be defined by $y = F(x)$, define new variables y^* and u_2^* by

$$y^* = y - F(x), \quad u_2^* = u_2 - F'(x), \tag{13}$$

where $F'(x) \equiv dF/dx$. The second of these is motivated by (11), which now gives $u_2^* = 0$ at the body suface $y^* = 0$.

Substituting (12) and (13) into (1)–(6) and retaining only leading-order terms gives equations for a thin body in a uniform hypersonic dusty-gas stream:

$$\frac{\partial \rho}{\partial x} + \frac{\partial}{\partial y^*}(\rho u_2^*) = 0, \tag{14}$$

$$\rho\left(\frac{\partial}{\partial x} + u_2^* \frac{\partial}{\partial y^*}\right) u_2^* + \rho F''(x) = -\frac{\partial p}{\partial y^*} + k_0 N_0 N (v_2 - u_2^* - F'(x)), \tag{15}$$

$$\rho\left(\frac{\partial}{\partial x} + u_2^* \frac{\partial}{\partial y^*}\right)(p/\rho) + (\gamma - 1) p \frac{\partial u_2^*}{\partial y^*} = D_1 N\{T_p/(\gamma M_\infty^2 \tau^2) - (p/\rho)\}, \tag{16}$$

$$\left[\frac{\partial}{\partial x} + (v_2 - F'(x)) \frac{\partial}{\partial y^*}\right] N + N \frac{\partial v_2}{\partial y^*} = 0, \tag{17}$$

$$\left[\frac{\partial}{\partial x} + (v_2 - F'(x)) \frac{\partial}{\partial y^*}\right] v_2 = -(k_0/m)(v_2 - u_2^* - F'(x)), \tag{18}$$

$$\left[\frac{\partial}{\partial x} + (v_2 - F'(x)) \frac{\partial}{\partial y^*}\right] T_p = -D_0(T_p - \gamma M_\infty^2 \tau^2 p/\rho), \tag{19}$$

where $k_0 \equiv k/U_\infty$, $N_0 \equiv N_\infty/\rho_\infty$, $D_0 \equiv D/(mcU_\infty)$, $D_1 \equiv DN_\infty/(c_v \rho_\infty U_\infty)$. It should be noted that the x-momentum equations in system (1)–(6) are decoupled and are therefore omitted from (14)–(19). The boundary conditions take a form identical with those of Van Dyke (1954) and hence are not repeated here. These equations and corresponding boundary conditions do not readily admit analytical solutions. The problem is further simplified by exploiting the ideas of Newtonian impact theory plus centrifugal force correction. Define, following Cole (1957), a basic small parameter for Newtonian theory by

$$\epsilon = (\gamma - 1)/(\gamma + 1). \tag{20}$$

We now perform the limiting process $\gamma \to 1$, $M_\infty \to \infty$ such that $H \equiv \epsilon^{-1} M_\infty^{-2} \tau^{-2}$ remains fixed. As is well known, this forces the shock to collapse (in theory) to the body surface, the shock layer being $O(\epsilon)$ in thickness. Hence, in order to examine this thin shock layer, the independent variable y^* must be stretched,

$$\tilde{y} = y^*/\epsilon. \tag{21}$$

The dependent flow variables are assumed to have the following asymptotic expansions:

$$\begin{aligned}
\rho(x, y^*; \gamma, M_\infty) &= (1/\epsilon)\,\rho^{(1)}(x, \tilde{y}; H) + \rho^{(2)}(x, \tilde{y}; H) + \ldots, \\
u_2^*(x, y^*; \gamma, M_\infty) &= \epsilon u_2^{(1)}(x, \tilde{y}; H) + \epsilon^2 u_2^{(2)}(x, \tilde{y}; H) + \ldots, \\
p(x, y^*; \gamma, M_\infty) &= p^{(1)}(x, \tilde{y}; H) + \epsilon p^{(2)}(x, \tilde{y}; H) + \ldots, \\
N(x, y^*; \gamma, M_\infty) &= N^{(1)}(x, \tilde{y}; H) + \epsilon N^{(2)}(x, \tilde{y}; H) + \ldots, \\
v_2(x, y^*; \gamma, M_\infty) &= \epsilon v_2^{(1)}(x, \tilde{y}; H) + \epsilon^2 v_2^{(2)}(x, \tilde{y}; H) + \ldots, \\
T_p(x, y^*; \gamma, M_\infty) &= T_p^{(1)}(x, \tilde{y}; H) + \epsilon T_p^{(2)}(x, \tilde{y}; H) + \ldots.
\end{aligned} \tag{22}$$

Expansions (22) are now substituted into (14)–(19) to obtain first- and second-order equations which approximate the small-disturbance equations.

(i) *First-order equations:*

$$\frac{\partial \rho^{(1)}}{\partial x} + \frac{\partial}{\partial \tilde{y}}(\rho^{(1)} u_2^{(1)}) = 0, \quad \frac{\partial p^{(1)}}{\partial \tilde{y}} = -F''(x)\,\rho^{(1)}, \tag{23), (24}$$

$$\left[\frac{\partial}{\partial x} + u_2^{(1)} \frac{\partial}{\partial \tilde{y}}\right][p^{(1)}/\rho^{(1)}] = 0, \tag{25}$$

$$\frac{\partial N^{(1)}}{\partial \tilde{y}} = \frac{\partial T_p^{(1)}}{\partial \tilde{y}} = 0, \quad \frac{\partial v_2^{(1)}}{\partial \tilde{y}} = -k_0/m. \tag{26), (27}$$

(ii) *Second-order equations:*

$$\frac{\partial \rho^{(2)}}{\partial x} + \frac{\partial}{\partial \tilde{y}}(\rho^{(1)} u_2^{(2)} + \rho^{(2)} u_2^{(1)}) = 0, \tag{28}$$

$$\left[\frac{\partial p^{(2)}}{\partial \tilde{y}} + F''\rho^{(2)}\right] = -k_0 N_0 F' N^{(1)} - \rho^{(1)}\left[\frac{\partial}{\partial x} + u_2^{(1)} \frac{\partial}{\partial \tilde{y}}\right] u_2^{(1)}, \tag{29}$$

$$\begin{aligned}
\rho^{(1)}\left[\frac{\partial}{\partial x} + u_2^{(1)} \frac{\partial}{\partial \tilde{y}}\right][p^{(2)}/\rho^{(1)} - p^{(1)}\rho^{(2)}/\rho^{(1)2}] + \rho^{(1)} u_2^{(2)} \frac{\partial}{\partial \tilde{y}}(p^{(1)}/\rho^{(1)}) \\
= D_1 N^{(1)}[H T_p^{(1)} - p^{(1)}/\rho^{(1)}] - 2p^{(1)} \frac{\partial u_2^{(1)}}{\partial \tilde{y}},
\end{aligned} \tag{30}$$

$$F' \frac{\partial N^{(2)}}{\partial \tilde{y}} = \frac{\partial N^{(1)}}{\partial x} + \frac{\partial}{\partial \tilde{y}}(N^{(1)} v_2^{(1)}), \tag{31}$$

$$F' \frac{\partial v_2^{(2)}}{\partial \tilde{y}} = \frac{\partial v_2^{(1)}}{\partial x} - (k_0/m)\, u_2^{(1)}, \tag{32}$$

$$F' \frac{\partial T_p^{(2)}}{\partial \tilde{y}} = \frac{\partial T_p^{(1)}}{\partial x} + D_0[T_p^{(1)} - p^{(1)}/H\rho^{(1)}]. \tag{33}$$

To obtain the shock conditions we follow Cole (1957) and assume an expansion for the shock shape in the form

$$\tilde{y} = g(x) + \epsilon g_1(x) + \ldots. \tag{34}$$

Then, the boundary conditions associated with systems (23)–(27) and (28)–(33), are:

(i) *First-order boundary conditions:*

$$p^{(1)}(x,g) = F'^2, \quad \rho^{(1)}(x,g) = F'^2/(H+F'^2), \quad u_2^{(1)}(x,g) = g' - F' - H/F',$$
$$N^{(1)}(x,g) = 1, \quad T_p^{(1)}(x,g) = 1, \quad v_2^{(1)}(x,g) = 0 \tag{35}$$

and

$$u_2^{(1)} = 0 \quad \text{at} \quad \tilde{y} = 0. \tag{36}$$

(ii) *Second-order boundary conditions:*

$$p^{(2)}(x,g) = 2F'g' - F'^2 - H - g_1 \frac{\partial p^{(1)}}{\partial \tilde{y}}(x,g),$$
$$\rho^{(2)}(x,g) = HF'(2g'+F')(H+F'^2)^{-2} - g_1 \frac{\partial \rho^{(1)}}{\partial \tilde{y}}(x,g),$$
$$u_2^{(2)}(x,g) = g_1' - g' + Hg'/F'^2 + H/F', \tag{37}$$
$$N^{(2)}(x,g) = -g_1 \frac{\partial N^{(1)}}{\partial \tilde{y}}(x,g), \quad T_p^{(2)}(x,g) = -g_1 \frac{\partial T_p^{(1)}}{\partial \tilde{y}}(x,g),$$
$$v_2^{(2)}(x,g) = -g_1 \frac{\partial v_2^{(1)}}{\partial \tilde{y}}(x,g),$$

and

$$u_2^{(2)} = 0 \quad \text{at} \quad \tilde{y} = 0. \tag{38}$$

4. Solution for the wedge

We now consider the solution of the Newtonian approximation equations (23)–(27) and (28)–(33) for the case of a wedge-shaped body, defined by $F(x) = x$.

(i) *First-order solution:* Equations (24), (26) and (27) immediately give the following solution variables satisfying conditions (35):

$$N^{(1)} = 1, \quad p^{(1)} = 1, \quad T_p^{(1)} = 1, \quad v_2^{(1)} = -(k_0/m)[\tilde{y} - g(x)].$$

To obtain the remaining unknown solutions and $g(x)$, equation (23) is used to introduce a stream function $\psi(x, \tilde{y})$ such that

$$\frac{\partial \psi}{\partial x} = -\rho^{(1)} u_2^{(1)}, \quad \frac{\partial \psi}{\partial \tilde{y}} = \rho^{(1)}. \tag{39}$$

Transforming to the Von Mises variables (x, ψ) to solve (25) and (23) for $\rho^{(1)}$ and $u_2^{(1)}$ and applying conditions (36) at $\tilde{y} = 0$ to obtain $g(x)$, the complete first-order solution is:

$$N^{(1)} = T_p^{(1)} = p^{(1)} = 1, \quad \rho^{(1)} = 1/(H+1),$$
$$u_2^{(1)} = 0, \quad v_2^{(1)} = (k_0/m)[(H+1)x - \tilde{y}], \quad g(x) = (H+1)x. \tag{40}$$

(ii) *Second-order solution:* The second-order approximation equations for the wedge are obtained by using solutions (40) in (28)–(33). These are:

$$\left.\begin{aligned}
&\frac{\partial \rho^{(2)}}{\partial x} + (H+1)^{-1} \frac{\partial u_2^{(2)}}{\partial \tilde{y}} = 0, \quad \frac{\partial p^{(2)}}{\partial \tilde{y}} = -k_0 N_0, \\
&\frac{\partial}{\partial x}\{p^{(2)} - (H+1)\rho^{(2)}\} = -D_1, \\
&\frac{\partial N^{(2)}}{\partial \tilde{y}} = -(k_0/m) = -(H+1)^{-1} \frac{\partial v_2^{(2)}}{\partial \tilde{y}}, \quad \frac{\partial T_p^{(2)}}{\partial \tilde{y}} = -D_0/H.
\end{aligned}\right\} \tag{41}$$

The associated boundary conditions (37), (38) are

$$\left.\begin{array}{l} N^{(2)} = T_2^{(2)} = 0, \quad v_2^{(2)} = (k_0/m)g_1(x), \\ p^{(2)} = H+1, \quad \rho^{(2)} = H(2H+3)(H+1)^{-2}, \quad u_2^{(2)} = g_1'+H^2+H-1 \end{array}\right\} \tag{42}$$

at $\tilde{y} = g(x) = (H+1)x$, and $u_2^{(2)} = 0$ at $\tilde{y} = 0$.

The solution of system (41) satisfying conditions (42) is:

$$\begin{gathered} N^{(2)} = (k_0/m)[(H+1)x-\tilde{y}], \quad T_{p}^{(2)} = (D_0/H)[(H+1)x-\tilde{y}], \\ v_2^{(2)} = (k_0/m)\{(H+1)\tilde{y}-H(2H+3)x-(H+1)[D_1+(H+1)k_0N_0]x^2/2\}, \\ p^{(2)} = k_0N_0[(H+1)x-\tilde{y}]+(H+1), \\ \rho^{(2)} = (H+1)^{-2}[D_1+k_0N_0(H+1)][(H+1)x-\tilde{y}]+H(2H+3)(H+1)^{-2}, \\ u_2^{(2)} = -[D_1+(H+1)k_0N_0]\tilde{y}, \\ g_1(x) = (1-H-H^2)x-(H+1)[D_1+(H+1)k_0N_0]x^2/2. \end{gathered} \tag{43}$$

The gas temperature can be found from (7) as

$$T = T'/T_\infty = (H+1)/H+\epsilon\{-H^2+H+3-D_1[(H+1)x-\tilde{y}]\}/H. \tag{44}$$

5. Fluid streamlines and dust-particle paths

The fluid streamlines and dust-particle paths are the solutions of the equations

$$\frac{dy'}{dx'} = u_2'/u_1' \quad \text{and} \quad \frac{dy'}{dx'} = v_2'/v_1'$$

respectively. In first-order hypersonic small-disturbance theory u_1' and v_1' are replaced by 1. In the non-dimensional variables (unprimed) these equations become, for the Newtonian approximation,

$$\frac{dy}{dx} = u_2 = F'+u_2^* = 1+\epsilon u_2^{(1)}+\epsilon^2 u_2^{(2)}+\dots$$

and

$$\frac{dy}{dx} = v_2 = \epsilon v_2^{(1)}+\epsilon^2 v_2^{(2)}+\dots. \tag{45}$$

With the results in (40) and (43), and recalling that $\epsilon\tilde{y} = y-F(x) = y-x$, the equations (45) may be integrated to give the equations for the fluid streamlines and the dust-particle paths to order ϵ^2 as

$$y = (x+A)-\epsilon A[D_1+(H+1)k_0N_0]x+\epsilon^2 A[D_1+(H+1)k_0N_0]^2x^2/2+\dots \quad \text{(fluid)} \tag{46}$$

and

$$\begin{aligned} y = {} & \left[x-\frac{m}{k_0}+B\exp\left\{-\frac{k_0}{m}x\right\}\right]+\epsilon(H+1)\left[x-\frac{2m}{k_0}+B\frac{k_0}{m}x\exp\left\{-\frac{k_0}{m}x\right\}\right] \\ & +\epsilon^2\left[-\frac{2m}{k_0}(H+1)^2+\left\{1-H-H^2+\frac{m}{k_0}(H+1)[D_1+(H+1)k_0N_0]\right\}\left\{x-\frac{m}{k_0}\right\}\right. \\ & \left.-\frac{(H+1)}{2}[D_1+(H+1)k_0N_0]x^2+B\left(\frac{k_0}{m}\right)^2\frac{(H+1)^2}{2}x^2\exp\left\{-\frac{k_0}{m}x\right\}\right]+\dots \quad \text{(dust).} \end{aligned} \tag{47}$$

In (46) and (47), A and B are parameters which vary from streamline to streamline and dust-particle path to path respectively.

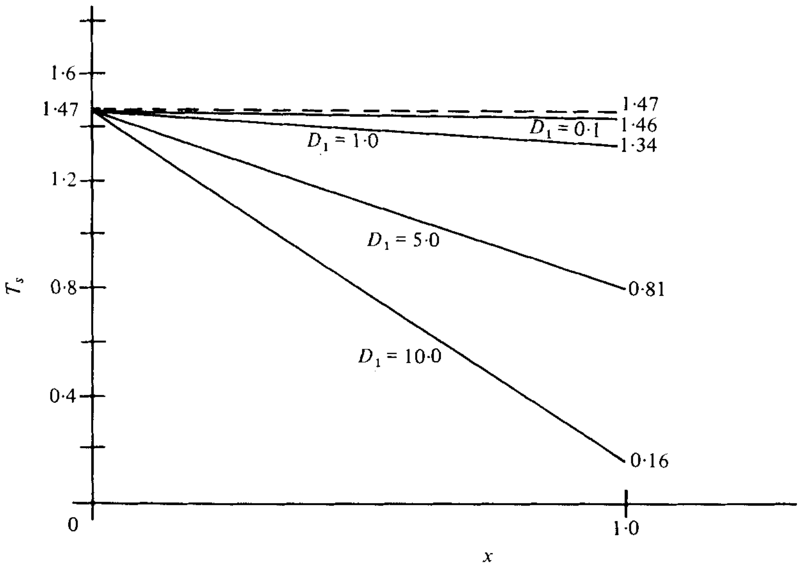

FIGURE 1. Gas temperature at wedge surface for various D_1; $\gamma = 1{\cdot}2$, $M_\infty^2\,\tau^2 = 5$. ---, clean-gas solution.

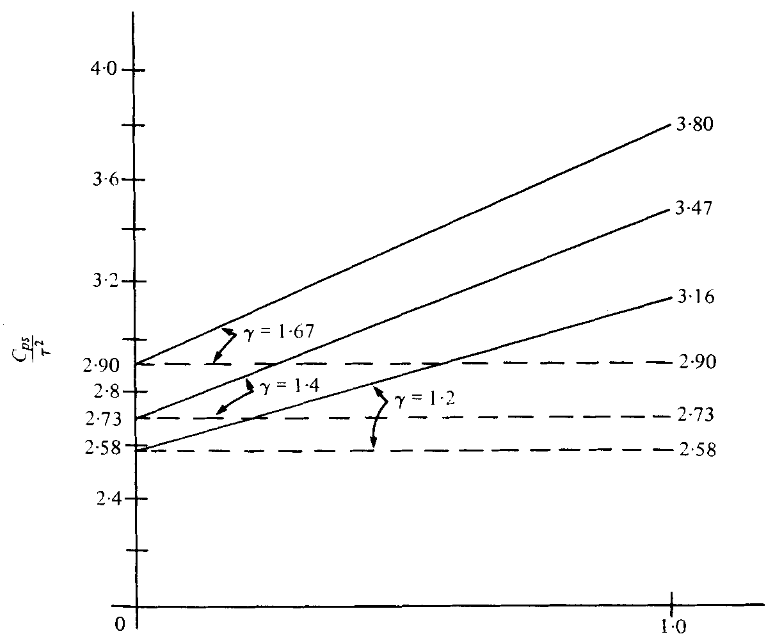

FIGURE 2. Surface pressure coefficient for various γ; $k_0 N_0 = 1$, $M_\infty^2\,\tau^2 = 5$. ---, clean-gas solution.

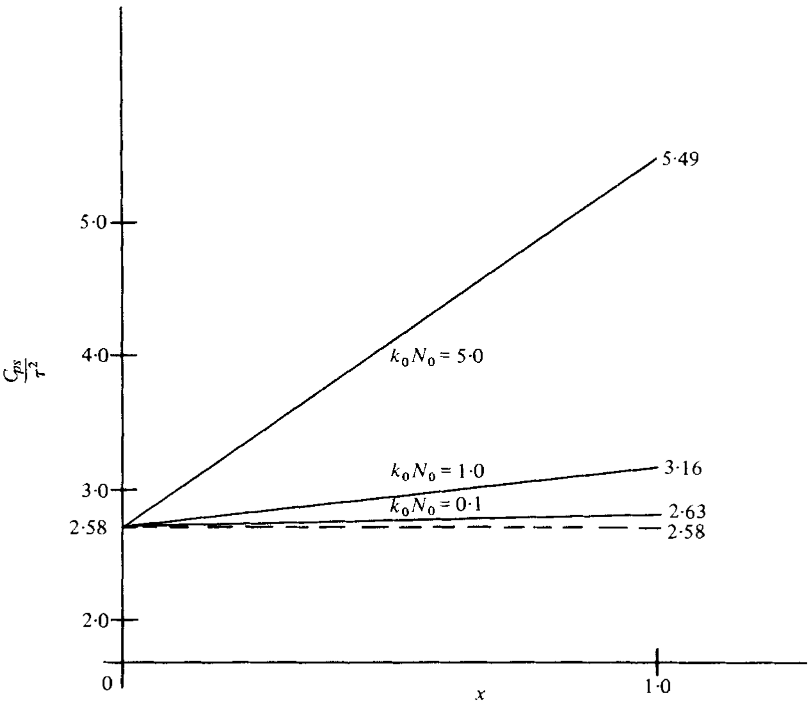

FIGURE 3. Surface pressure coefficient for various k_0N_0; $\gamma = 1·2$, $M_\infty^2\,\tau^2 = 5$. – – –, clean-gas solution.

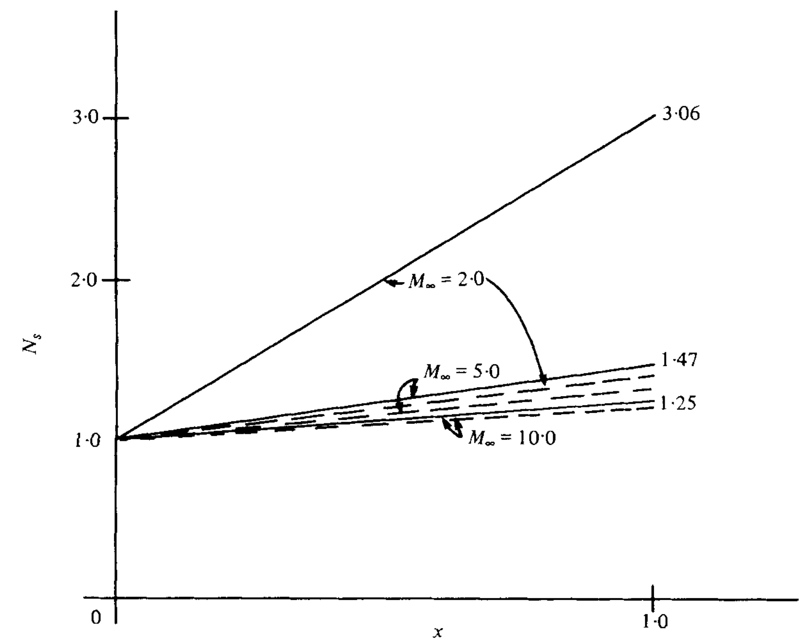

FIGURE 4. Particle number density at wedge surface for various M_∞; $\tau = \tan^{-1} 20°$, $\gamma = 1·4$, $k_0/m = 1$. —— present theory; – – –, Peddieson (1975).

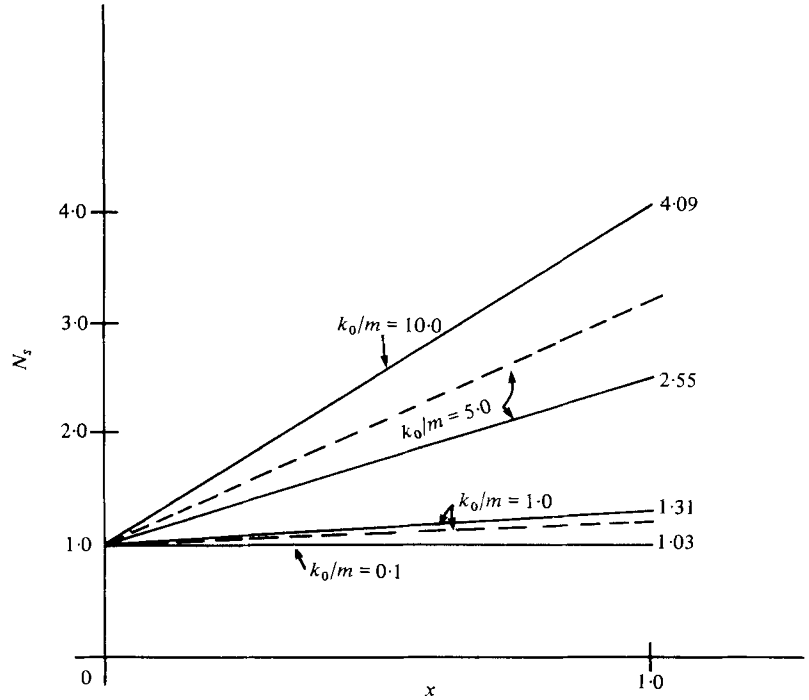

FIGURE 5. Particle number density at wedge surface for various k_0/m; $M_\infty = 10$, $\tau = \tan^{-1} 15°$, $\gamma = 1{\cdot}4$. ——, present theory; – – –, Peddieson (1975).

6. Discussion of results

Some typical calculations based on the preceding theory and solutions (40), (43) and (44) are presented graphically in figures 1–6.

(i) *Gas quantities:* Figure 1 shows the variation of gas temperature along the wedge surface for various values of the interphase heat-transfer coefficient D_1. No comparative data seems to be available since earlier researchers have neglected the influence of the dust particles on the gas flow. The clean-gas solution (in the second-order Newtonian theory) is indicated for comparison. It is seen that the gas temperature decreases along the wedge surface and at a greater rate as D_1 increases.

Figure 2 plots the surface pressure coefficient, given by

$$C_{ps} = 2\tau^2 p_s + \ldots = 2\tau^2\{1+\epsilon(H+1)(1+k_0N_0x)\}+\ldots$$

for various γ. Again, corresponding clean gas solutions are given for comparison. It is observed that C_{ps} increases as γ increases. The results are for the case $k_0N_0 = 1$ (momentum transfer coefficient from dust to gas phase) and the most reliable data is for $\gamma = 1{\cdot}2$ (since the present theory assumes $\gamma \to 1$). For this case it is seen that the effect of the presence of the dust is to increase the pressure on the wedge by up to 22·5 % (at $x = 1$). Figure 3 shows that C_{ps} increases rapidly with increasing effectiveness of the interphase momentum transfer coefficient k_0N_0.

(ii) *Dust quantities:* A comparison between the present theory and the results of Peddieson (1975), where the dust effect on the gas flow is neglected, is shown in figure 4. The particle number density at the wedge surface is plotted for various M_∞ and for a

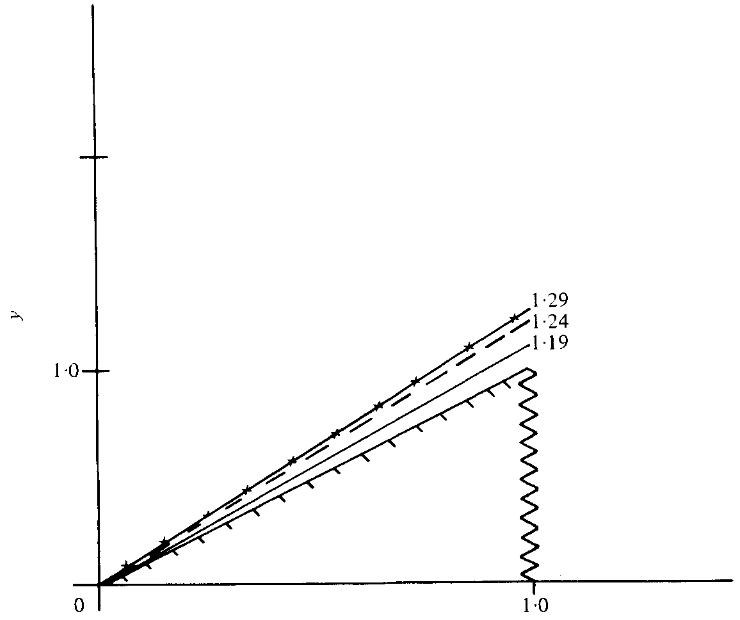

FIGURE 6. Typical shock wave location; $\gamma = 1{\cdot}2$, $M_\infty^2\,\tau^2 = 5$, $D_1 = 1$, $k_0 N_0 = 1$. –×–, Newtonian first-order clean and dusty gas; – – –, Newtonian second-order clean gas; ——, Newtonian second-order dusty gas.

wedge half-angle of 20 degrees. Agreement with previous results is good for large M_∞ (for which the present theory applies).

The variation of N along the surface for various k_0/m is illustrated in figure 5 for $M_\infty = 10$ and $\tau = \tan 15°$. Comparison with earlier results is good for small values of k_0/m. For larger k_0/m, Peddieson's results may not be valid since, as k_0/m increases, the particle phase becomes more compressed and hence the particle-phase mass fraction may not be negligible near the surface of the body.

(iii) *Shock location:* Lastly, figure 6 shows the predicted shock wave location in comparison to the clean gas location. The effect of the dust is to bend the shock wave and push it towards the wedge surface. It is seen from (43) that increasing the influence of the dust on the gas (through D_1 and $k_0 N_0$) has the effect of forcing the shock closer to the surface of the wedge.

7. Conclusions

The Newtonian theory for hypersonic flow of a dusty gas past a slender body is developed up to second order in $\epsilon = (\gamma - 1)/(\gamma + 1)$. First- and second-order equations are solved for the wedge, analytical expressions are obtained for all flow variables and the shock wave location. Streamlines and dust-particle paths are also obtained. The variation of important aerodynamical quantities along the wedge surface for various combinations of the physical and thermodynamical parameters, such as the adiabatic exponent and interphase momentum and heat transfer coefficients, are depicted in graphical form. The presence of the dust increases the pressure and decreases the gas temperature along the wedge surface and forces the shock wave to bend towards the body. It is also seen that the particle number density decreases for increasing M_∞ and increases for increasing interphase momentum transfer coefficient.

As a final note, it must be pointed out that from the inviscid viewpoint presented here the flow is correctly predicted at the nose of the wedge. However, from a more realistic viewpoint one expects a stagnation point at the nose. This might be handled by the addition of a viscous boundary layer on the wedge.

REFERENCES

Barron, R. M. & Wiley, J. T. 1979 Remarks on dusty hypersonic wedge flow. *A.I.A.A. J.* **17**, 317–318.

Carrier, G. F. 1958 Shock waves in a dusty gas. *J. Fluid Mech.* **4**, 376–382.

Cole, J. D. 1957 Newtonian flow theory for slender bodies. *J. Aero. Sci.* **24**, 448–455.

Marble, F. E. 1970 Dynamics of dusty gases. *Ann. Rev. Fluid Mech.* **2**, 397–446.

Peddieson, J. 1975 Two-phase flow in supersonic shock layers. *Developments in Mechanics*, vol. 8, *Proc. 14th Midwestern Mech. Conf.*, pp. 467–482.

Peddieson, J. & Lyu, C.-H. 1973 Dusty hypersonic wedge flow. *A.I.A.A. J.* **11**, 110–112.

Peddieson, J. & Lyu, C.-H. 1974 Supersonic two-phase flow past a cone. *Proc. 10th Southeastern Seminar on Thermal Sciences.*

Probstein, R. F. & Fassio, F. 1970 Dusty hypersonic flows. *A.I.A.A. J.* **8**, 772–779.

Spurk, J. H. & Gerber, N. 1972 Dust collection efficiency for power law bodies in hypersonic flight. *A.I.A.A. J.* **10**, 755–761.

Van Dyke, M. D. 1954 A study of hypersonic small-disturbance theory. *N.A.C.A. Tech. Note* 1194.

Waldman, G. D. & Reinecke, W. G. 1971 Particle trajectories, heating and breakup in hypersonic shock layers. *A.I.A.A. J.* **9**, 1040–1048.